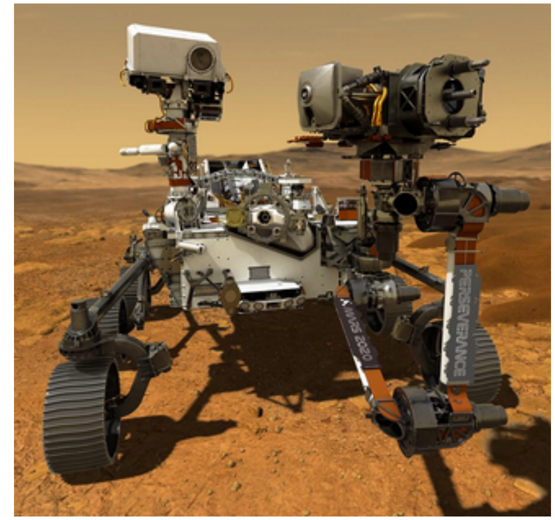
Chapter 5
Rational Exponents and Radical Functions



1. nth Roots and Rational Exponents
2. **Properties of Rational Exponents and Radicals**
3. Graphing Radical Functions
4. Solving Radical Equations and Inequalities
5. Performing Function Operations
6. Inverse of a Function

5.2 - Properties of Rational Exponents and Radicals

Properties of Rational Exponents

Let a and b be real numbers and let m and n be rational numbers, such that the quantities in each property are real numbers.

Property Name	Definition	Example
Product of Powers	$a^m \cdot a^n = a^{m+n}$	$5^{1/2} \cdot 5^{3/2} = 5^{(1/2+3/2)} = 5^2 = 25$
Power of a Power	$(a^m)^n = a^{mn}$	$(3^{5/2})^2 = 3^{(5/2 \cdot 2)} = 3^5 = 243$
Power of a Product	$(ab)^m = a^m b^m$	$(16 \cdot 9)^{1/2} = 16^{1/2} \cdot 9^{1/2} = 4 \cdot 3 = 12$
Negative Exponent	$a^{-m} = \dfrac{1}{a^m}, a \neq 0$	$36^{-1/2} = \dfrac{1}{36^{1/2}} = \dfrac{1}{6}$
Zero Exponent	$a^0 = 1, a \neq 0$	$213^0 = 1$
Quotient of Powers	$\dfrac{a^m}{a^n} = a^{m-n}, a \neq 0$	$\dfrac{4^{5/2}}{4^{1/2}} = 4^{(5/2-1/2)} = 4^2 = 16$
Power of a Quotient	$\left(\dfrac{a}{b}\right)^m = \dfrac{a^m}{b^m}, b \neq 0$	$\left(\dfrac{27}{64}\right)^{1/3} = \dfrac{27^{1/3}}{64^{1/3}} = \dfrac{3}{4}$

5.2 - Properties of Rational Exponents and Radicals

Rational Exponents

$$5^2 = 25 \qquad 5^{-2} = \frac{1}{25} \qquad 5^0 = 1$$

$$5^{1/2} = \sqrt{5} \qquad 5^{\frac{2}{3}} = \left(5^{\frac{1}{3}}\right)^2 = \left(\sqrt[3]{5}\right)^2 = \left(5^2\right)^{\frac{1}{3}} = \sqrt[3]{5^2}$$

Practice

$a.\ 16^{\frac{3}{4}}$ 8

$b.\ 25^{\frac{-3}{2}}$ $\frac{1}{125}$

$c.\ 9^{2.5}$ 243

5.2 - Properties of Rational Exponents and Radicals

Negative Exponents

$$(2^2)^{-3} \qquad \frac{1}{64}$$

$$\frac{3x^{-3}y^{-2}z^4}{2^{-2}x^2y^{-3}z^{-1}} \qquad \frac{12yz^5}{x^5}$$

Practice

$a.\ (3^{-1})^{-2}$ $b.\ \dfrac{10^{-3}}{10^{-5}}$ $c.\ \dfrac{x^{-8}y^2}{x^{-5}y^{-2}}$

9 100 $\dfrac{y^4}{x^3}$

5.2 - Properties of Rational Exponents and Radicals

Write in exponential form

$$\sqrt[3]{\frac{a^5b^3}{c^2}} \qquad a^{\frac{5}{3}}bc^{\frac{-2}{3}}$$

$$\left(\frac{1}{\sqrt[3]{4}}\right)^{\frac{-3}{2}} \qquad 2$$

Practice

$a.$ $\sqrt[4]{\frac{16^3 \cdot a^{-2}}{b^6}}$ $\quad 8a^{-\frac{1}{2}}b^{-\frac{3}{2}}$

$b.$ $\sqrt[5]{\frac{\sqrt[3]{x^4}\sqrt[4]{y}}{\sqrt[3]{9^2}}}$ $\quad 3^{-\frac{4}{15}}x^{\frac{4}{15}}y^{\frac{1}{20}}$

5.2 - Properties of Rational Exponents and Radicals

Express in simplest radical form

$$\sqrt[3]{4} \cdot \sqrt[3]{4} \qquad 2\sqrt[3]{2}$$

$$\frac{\sqrt[3]{4}}{\sqrt[6]{2}} \qquad \sqrt{2}$$

Practice

$a.\ \dfrac{\sqrt[5]{27^3}}{\sqrt[5]{9^2}}$ $\quad 3$

$b.\ \sqrt[6]{8^3} \div \sqrt[6]{4^2}$ $\quad \sqrt[6]{2^5}$

5.2 - Properties of Rational Exponents and Radicals

Express in simplest form

$$\left(\frac{x^{-2}}{yz^{-3}}\right)^{-2} \cdot \left(\frac{-2z^{-2}}{y^2x^0}\right)^{-3}$$

$$\frac{-x^4y^8}{8}$$

$$\left(\frac{x+1}{yz^2}\right)^{-2} \cdot \left(\frac{-x^{a+b}}{y^{-a}}\right)^{2}$$

$$\frac{x^{2a+2b} \cdot y^{2a+2} \cdot z^4}{x^2+2x+1}$$

5.2 - Properties of Rational Exponents and Radicals

Adding and Subtracting Radicals

$$3\sqrt{2} + 5\sqrt{2} \quad = 8\sqrt{2}$$

$$\sqrt{8} + 2\sqrt{3} + 3\sqrt{32} \quad = 14\sqrt{2} + 2\sqrt{3}$$

Practice

$a.\ 3\sqrt{2} - \sqrt{2} + 2\sqrt{2}$

$4\sqrt{2}$

$b.\ 5\sqrt{3} + 2\sqrt{27} - 3\sqrt{4}$

$11\sqrt{3} - 6$

$c.\ 3\sqrt{343} + 2\sqrt{49} + 5\sqrt{7}$

$26\sqrt{7} + 14$

5.2 - Properties of Rational Exponents and Radicals

Fractional Exponents

$$a^{\frac{1}{2}} \cdot a^{\frac{3}{2}} = a^{\frac{1}{2}+\frac{3}{2}} = a^2$$

$$\frac{3^{\frac{2}{3}}}{3^{\frac{1}{6}}} = 3^{\frac{2}{3}-\frac{1}{6}} = 3^{\frac{1}{2}}$$

Practice

$a.\ 2^{\frac{3}{4}} \cdot 2^{\frac{1}{2}}$ $\quad 2^{\frac{5}{4}}$

$b.\ \dfrac{3}{3^{\frac{1}{4}}}$ $\quad 3^{\frac{3}{4}}$

$c.\ \left(\dfrac{20^{\frac{1}{2}}}{5^{\frac{1}{2}}}\right)^3$ $\quad 8$

$d.\ \left(5^{\frac{1}{3}} \cdot 7^{\frac{1}{4}}\right)^3$ $\quad 5 \cdot 7^{\frac{3}{4}}$

5.2 - Properties of Rational Exponents and Radicals

Simplify the expression

$$\sqrt{x} \cdot \sqrt[3]{x} \cdot \sqrt[6]{x} \qquad x$$

$$\sqrt[4]{x} \cdot (\sqrt[6]{x})^2 \div \sqrt[3]{x} \qquad \sqrt[4]{x}$$

Practice

$a.\ \left(\left(b^{\frac{1}{2}}\right)^{\frac{-2}{3}}\right)^{\frac{3}{4}}$ $\qquad b^{-\frac{1}{4}}$

$b.\ a^{\frac{1}{2}}\left(a^{\frac{3}{2}} - 2a^{\frac{1}{2}}\right)$ $\qquad a^2 - 2a$

$c.\ \left(x^{\frac{3}{2}} - 2x^{\frac{5}{2}}\right) \div x^{\frac{1}{2}}$ $\qquad x - 2x^2$

5.2 - Properties of Rational Exponents and Radicals

Simplify the expression

$$\frac{1}{5}\sqrt{w}+\frac{3}{5}\sqrt{w} \qquad \frac{4}{5}w$$

Practice

$$a.\ \ 12\sqrt[3]{2z^5}-z\sqrt[3]{54z^2} \qquad b.\ \ \sqrt{9w^5}-w\sqrt{w^3}$$

$$9z\sqrt[3]{2z^2} \qquad 2w^2\sqrt{w^3}$$

5.2 - Properties of Rational Exponents and Radicals

Solve the equation

$\sqrt{x} = 5$ $\quad x = 25$

$(3x+1)^{\frac{3}{4}} = 8$ $\quad x = 5$

Practice

$a.\ \ x^2 = 16$

$x = \pm 4$

$b.\ \ \sqrt{2x} + 2 = 10$

$x = 32$

$c.\ \ \sqrt{2x} + 12 = 10$

$no\ solution$

$d.\ \ (8-y)^{\frac{1}{3}} = 4$

$y = -56$

$e.\ \ (3n-1)^{-\frac{2}{3}} = \dfrac{1}{4}$

$n = 3, -\dfrac{7}{3}$

$f.\ \ (x^2+9)^{\frac{1}{2}} = 5$

$x = \pm 4$

5.2 - Properties of Rational Exponents and Radicals

Binomials with Radicals

$$\frac{2}{\sqrt{2}-3} \cdot \frac{\sqrt{2}+3}{\sqrt{2}+3} = \frac{2\sqrt{2}+6}{2-9} = \frac{-2\sqrt{2}-6}{7}$$

Practice

$a.\ \dfrac{4}{\sqrt{3}+2}$

$-4\sqrt{3}+8$

$b.\ \dfrac{\sqrt{3}}{2\sqrt{5}-1}$

$\dfrac{2\sqrt{15}+\sqrt{3}}{19}$

$c.\ \dfrac{2+\sqrt{3}}{3\sqrt{2}-1}$

$\dfrac{3\sqrt{6}+6\sqrt{2}+\sqrt{3}+2}{17}$

5.2 - Properties of Rational Exponents and Radicals

Binomials with Radicals

$$\frac{3}{\sqrt[3]{z}+3}$$

$$(a+b)(a^2-ab+b^2)=a^3+b^3$$

$$(\sqrt[3]{z}+3)(z^{2/3}-3\sqrt[3]{z}+9)=z+27$$

$$\frac{3}{\sqrt[3]{z}+3}\cdot\frac{(z^{2/3}-3\sqrt[3]{z}+9)}{(z^{2/3}-3\sqrt[3]{z}+9)}=\frac{3z^{2/3}-9\sqrt[3]{z}+27}{z+27}$$